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Binary Third degree Diophantine Equation

$$5(x-y)^3 = 8xy$$

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Abstract

This article empathize on finding non-zero different integer solutions to binary third degree diophantine equation $5(x-y)^3 = 8xy$. Two different sets of solutions in integers are presented. Some fascinating relations from the solutions are obtained. The method to get second order Ramanujan numbers is illustrated.

Keywords : Non-homogeneous cubic , Binary cubic ,integer solutions , Ramanujan numbers

Notations :

$$P_n^5 = \frac{n^2(n+1)}{2}$$

$$t_{m,n} = \frac{n[2+(n-1)(m-2)]}{2}$$

$$Ct_{6,n} = 3n^2 + 3n + 1$$

$$Th_n = 3*2^n - 1$$

$$M_n = 2^n - 1$$

$$CP_k^{14} = \frac{7k^3 - 4k}{3}$$

$$CP_k^{15} = \frac{15k^3 - 9k}{6}$$

$$CP_k^{16} = \frac{8k^3 - 5k}{3}$$

Introduction

The third degree Diophantine equations are enormous in variety and they have contributed to expansion of research in this filed[1,2]. For a extensive approach of these types of problems , one may refer [3-28]. In this article a search is made to get solutions in integers for the considered problem through different methods and also the method of getting second order Ramanujan numbers from the obtained solution is discussed. Some fascinating relations from the solutions are presented.

Method of analysis

The considered non-homogeneous binary third degree equation under is

$$5(x-y)^3 = 8xy \tag{1}$$

Taking

$$x = u + v\ , y = u - v, u \neq v \neq 0 \tag{2}$$

Substituting (2) in (1) , we get

$$u^2 = v^2 (1+5v) \tag{3}$$

which satisfies

$$v = k(5k+2)\ , u = k(5k+1)(5k+2) \tag{4}$$

Substituting (4) in (2) , one has

$$x = x(k) = k(5k+2)^2\ , y = y(k) = 5k^2(5k+2) \tag{5}$$

Observe that (5) satisfies (1).

A few numerical values for the obtained solution to equation (1) are presented in Table 1 below:

Table 1-Numerical values

k	x(k)	y(k)
1	49	35
2	288	240
3	3*289	17*45
4	4*484	22*80
5	5*729	27*125

From the above Table 1, it is seen that both the values of $x(k), y(k)$ are alternatively odd and even.

Relations among the solutions :

1. $5[2y(k) - x(k) + 4k]$ is a cubical integer
2. $k(2y(k) - x(k))$ is written as difference of two squares
3. $k(2x(k) - y(k))$ is written as difference of two squares
4. $x(k) - y(k) - 2Ct_{6,k} + 2k + 2$ is a perfect square
5. $x(k) - y(k) - 13k = t_{22,k}$

6. $\sum_{k=1}^{n}[x(k)-y(k)]=\frac{20P_n^5+22t_{3,n}}{3}$

7. $\sum_{k=1}^{n}y(k)=\frac{20t_{3,n}+189P_n^5+t_{3,n}*t_{152,n}}{6}$

8. $25xy$ is a cubical integer

9. $25k^3x(k)=(y(k))^2$

10. $x(2^n)-y(2^n)=Th_{2n}+M_{n+2}+7M_{2n}+9$

11. $x(k)-y(k)$ is a perfect square when k takes the values

$$k=k_n=\frac{(-1)^n\beta_{n+1}-2}{10},n=-1,0,1,2,...$$

where

$$\beta_{n+1}=19f_n+6\sqrt{10}g_n,$$
$$f_n=(19+6\sqrt{10})^{n+1}+(19-6\sqrt{10})^{n+1},$$
$$g_n=(19+6\sqrt{10})^{n+1}-(19-6\sqrt{10})^{n+1}$$

16. $[x(k+2)-y(k+2)]-2[x(k+1)-y(k+1)]+[x(k)-y(k)]=20$

17. $[x(k+4)-y(k+4)]-2[x(k+3)-y(k+3)]+2[x(k+2)-y(k+2)]-2[x(k+1)-y(k+1)]+[x(k)-y(k)]=40$

18. $[x(n+k)-y(n+k)]-[x(n+k-1)-y(n+k-1)]=20(k+n)-6$

19. $y(k)=20P_k^5+6CP_k^{15}+9k$

20. $y(k)=20P_k^5+3CP_k^{16}+3CP_k^{14}+9k$

Formulation of Second order Ramanujan numbers :

From each of the solutions of (1) given by (5) , one can find Second order Ramanujan numbers with base numbers as real integers .

Illustration 1

Cosider

$$
\begin{aligned}
y(k) &= 5k^2(5k+2) \\
&= 5k^2 * (5k+2) = (5k^2+2k)*5k \\
&= A*B = C*D \text{ say}
\end{aligned}
$$

From the above relation , one may observe that

$$
\begin{aligned}
&(A+B)^2+(C-D)^2 = (A-B)^2+(C+D)^2 = A^2+B^2+C^2+D^2 \\
&(5k^2+5k+2)^2+(5k^2-3k)^2 = (5k^2-5k-2)^2+(5k^2+7k)^2 \\
&\qquad = 50k^4+20k^3+54k^2+20k+4
\end{aligned}
$$

Thus , $50k^4+20k^3+54k^2+20k+4$ represents the second order Ramanujan number.

Illustration 2

Cosider

$$
\begin{aligned}
x(k) &= k(5k+2)^2 \\
&= (5k^2+2k)*(5k+2) = (5k+2)^2 * k \\
&= A*B = E*F \text{ say}
\end{aligned}
$$

In this case ,the corresponding Second order Ramanujan number is found to be

$$650k^4+1020k^3+630k^2+180k+20$$

Remark :

In addition to the solutions (5) , we have an another set of solutions in integers to (1)

given by

$$x = x(k) = 5k(5k^2 - 2k), y = y(k) = k(5k-2)^2$$

Conclusion

This article gives an approach to solve third degree equation with two unknowns though different methods to get solutions in integers. The research in this field may attempt to find various other methods to solve binary cubic equation and also approach to get second order Ramanujan numbers and find various other relation from the obtained solution.

References

[1] L.E. Dickson, History of Theory of Numbers, Vol.2, Chelsea Publishing Company, NewYork, 1952.

[2] L.J. Mordell, Diophantine equations, Academic press, New York, 1969.

[3] M.A. Gopalan, G. Sangeetha, "On the ternary cubic Diophantine equation $y^2 = Dx^2 + z^3$" , Archimedes J.Math 1(1), 2011, 7-14.

[4] M.A. Gopalan, B. Sivakami, "Integral solutions of the ternary cubic equation $4x^2 - 4xy + 6y^2 = ((k+1)^2 + 5)w^3$", Impact J.Sci.Tech, 6(1), 2012, 15-22.

[5] M.A. Gopalan, B. Sivakami, "On the ternary cubic Diophantine equation $2xz = y^2(x+z)$" , Bessel J.Math, 2(3), 2012, 171-177.

[6] M.A. Gopalan, S. Vidhyalakshmi, G. Sumathi, "On the homogeneous cubic equation with four unknowns $X^3 + Y^3 = 14Z^3 - 3W^2(X+Y)$", Discovery, 2(4), 2012, 17-19.

[7] S. Vidhyalakshmi, M.A. Gopalan, A. Kavitha, "Observation on homogeneous cubic equation with four unknowns $X^3 + Y^3 = 7^{2n} ZW^2$", IJMER, 3(3), May-June 2013, 1487-1492.

[8] S. Vidyalakshmi, T.R. Usharani, M.A. Gopalan, "Integral solutions of non-homogeneous ternary cubic equation $ax^2 + by^2 = (a+b)z^3$ ", Diophantus J.Math 2(1), 2013, 31-38.

[9] M.A. Gopalan, K. Geetha, "On the ternary cubic Diophantine equation $x^2 + y^2 - xy = z^3$ ", Bessel J.Math., 3(2), 2013,119-123.

[10] M.A. Gopalan, S. Vidhyalakshmi, A.Kavitha "Observations on the ternary cubic equation $x^2 + y^2 + xy = 12z^3$ ", Antartica J.Math, 10(5), 2013, 453-460.

[11] M.A. Gopalan, S. Vidhyalakshmi, K. Lakshmi, "Lattice points on the non-homogeneous cubic equation $x^3 + y^3 + z^3 + (x+y+z) = 0$ ", Impact J.Sci.Tech, 7(1), 2013, 21-25.

[12] M.A. Gopalan, S. Vidhyalakshmi, K. Lakshmi "Lattice points on the non-homogeneous cubic equation $x^3 + y^3 + z^3 - (x+y+z) = 0$ ", Impact J.Sci.Tech, 7(1), 2013, 51-55,

[13] M.A. Gopalan, S. Vidhyalakshmi, S. Mallika, "On the ternary non-homogenous cubic equation $x^3 + y^3 - 3(x+y) = 2(3k^2 - 2)z^3$", Impact J.Sci.Tech, 7(1), 2013, 41-45.

[14] M.A. Gopalan, S. Vidhyalakshmi, N. Thiruniraiselvi, "On homogeneous cubic equation with four unknowns $(x+y+z)^3 = z(xy + 31w^2)$ ", Cayley J.Math, 2(2), 2013, 163-168.

[15] M.A. Gopalan, S. Vidhyalakshmi, N. Thiruniraiselvi, "On homogeneous cubic equation with four unknowns $x^3 + y^3 = 21zw^2$", Review of Information Engineering and Applications, 1(4), 2014, 93-101.

[16] S. Vidhyalakshmi, M.A. Gopalan, S. Aarthy Thangam, "On the ternary cubic Diophantine equation $4(x^2 + x) + 5(y^2 + 2y) = -6 + 14z^3$ " International Journal of Innovative Research and Review (JIRR), 2(3), July-Sep 2014, 34-39.

[17] M.A. Gopalan, S. Vidhyalakshmi, E. Premalatha, C. Nithya, "On the cubic equation with four unknowns $x^3+y^3=31(k^2+3s^2)zw^2$", IJSIMR, 2(11), Nov-2014, 923-926.

[18] S. Vidhyalakshmi, T.R. Usha Rani, M.A. Gopalan, V. Kiruthika, "On the cubic equation with four unknowns $x^3+y^3=14zw^2$", IJSRP, 5(3), March 2015, 1-11.

[19] M.A. Gopalan, S. Vidhyalakshmi, J. Shanthi, "On the cubic equation with four unknowns $x^3+4z^3=y^3+4w^3+6(x-y)^3$", International Journal of Mathematics Trends and Technology, 20(1), April 2015, 75-84.

[20] M.A. Gopalan, N. Thiruniraiselvi and V. Kiruthika, "On the ternary cubic diophantine equation $7x^2-4y^2=3z^3$", IJRSR, 6(9), Sep-2015, 6197-6199.

[21] M.A. Gopalan, S. Vidhyalakshmi, J. Shanthi, J. Maheswari, "On ternary cubic diophantine equation $3(x^2+y^2)-5xy+x+y+1=12z^3$", International Journal of Applied Research, 1(8), 2015, 209-212.

[22] R. Anbuselvi, K. Kannaki, "On ternary cubic diophantine equation $3(x^2+y^2)-5xy+x+y+1=15z^3$", IJSR, 5(9), Sep 2016, 369-375.

[23] G. Janaki, C. Saranya, "Integral solutions of the ternary cubic equation $3(x^2+y^2)-4xy+2(x+y+1)=972z^3$", IRJET, 4(3), March 2017, 665-669.

[24] R. Anbuselvi, K.S. Araththi, "On the cubic equation with four unknowns $x^3+y^3=24zw^2$", IJERA, 7(11) (Part-I), Nov-2017, 01-06.

[25] Sharadha Kumar, M.A. Gopalan, ON THE CUBIC EQUATION $x^3+y^3+6(x+y)z^2=4w^3$, JETIR ,6(1), 2019, 658-660.

[26] S. Vidhyalakshmi, T. Mahalakshmi, M.A. Gopalan, "A Search On the Integer Solutions of Cubic Diophantine Equation with Four Unknowns

$x^3 - y^3 = 4(w^3 - z^3) + 3(x-y)^3$ ", International Journal of Engineering And Science, 10(8), August 2020, 13-17.

[27] S. Vidhyalakshmi, T. Mahalakshmi, M.A.Gopalan and G. Dhanalakshmi, "On homogeneous Cubic Equation with Four Unknowns $(x^3 + y^3) = 7zw^2$ " Stochastic Modeling & Applications, 25(3), Special Issue III, Part-3, July-December 2021, 1968-1978.

[28] S. Vidhyalakshmi, T. Mahalakshmi and M.A. Gopalan, "A Search On The Integer Solutions of Cubic Diophantine Equation with Four Unknowns $x^2 + y^2 + 4(35z^2 - 4 - w^2) = 6xyz$", International Journal of Grid and Distributed Computing, 13(2), 2021, 2581-2584.